USN | | | | | | | | | | |

10MAT41

Fourth Semester B.E. Degree Examination, June/July 2018

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, selecting at least TWO full questions from each part.***
2. ***Use of statistical tables is permitted.***

PART – A

1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = xy + y^2$, y(0) = 1 at x = 0.1 and x_2 = 0.2. **(06 Marks)**

b. Obtain an approximate solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial conditions y = 1 at x = 0 for the range $0 \le x \le 0.4$ in steps of 0.2, using Euler's modified method. Perform two modifications at each step. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04 and y(0.3) = 2.09, find y(0.4) correct to five decimal places by employing the Milne's predictor-correct method. Use corrector formula twice. **(07 Marks)**

2 a. Find an approximate value of y and z corresponding to x = 0.1 given that y(0) = 2, z(0) = 1 and $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$. Using Picard's method. **(06 Marks)**

b. Solve, $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2, correct to four decimal places, with initial conditions x = 0, y = 1, $\frac{dy}{dx} = 0$, using Runge-Kutta method. **(07 Marks)**

c. Obtain an approximate solution at the point x = 0.4 of the initial value problem, $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, y(0) = 1, y′(0) = 0.1 using Milner's method. Given y(0) = 1, y(0.1) = 1.03995, y(0.2) = 1.138036, y(0.3) = 1.29865, y′(0) = 0.1, y′(0.1) = 0.6955, y′(0.2) = 1.258, y′(0.3) = 1.873. **(07 Marks)**

3 a. If f(z) = u + iv is an analytic function, then prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$. **(06 Marks)**

b. Find an analytic function f(z) = u + iv, given that $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. **(07 Marks)**

c. Find an analytic function f(z) = u +iv given the imaginary part $v = r^2\cos 2\theta - r\cos\theta + 2$. **(07 Marks)**

4 a. Find the bilinear transformation that transforms the points z_1 = i, z_2 = 1, z_3 = -1 onto the points w_1 = 1, w_2 = 0, w_3 = ∞ respectively. **(06 Marks)**

b. Evaluate $I = \int_{z=0}^{2+i} (\bar{z})^2 dz$ along the following curves:

i) The straight line $y = \frac{x}{2}$ from the origin θ to the point B(2 + i).

ii) The real axis from 0 to 2 and then vertically to 2 + i. **(07 Marks)**

c. State and prove Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Obtain the series solution Bessel's differential equation leading to Bessel's function of first kind. **(08 Marks)**

b. If α and β are distinct roots of the equation $J_n(ax) = 0$, then prove that $\int_0^a x J_n(\alpha x).J_n(\beta x)dx = 0$. **(07 Marks)**

c. Evaluate $p_0(x)$, $p_1(x)$, $p_2(x)$, $p_3(x)$ by using the Rodrigue's formula. **(05 Marks)**

6 a. A husband and wife appear for two vacancies of a post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that (i) both of them will be selected? (ii)Only one of them is selected? (iii) Neither is selected? **(06 Marks)**

b. What are independent events? If A and B are independent prove that (i) A and $\overline{B}$ are independent, (ii) $\overline{A}$ and B are independent and (iii) $\overline{A}$ and $\overline{B}$ are independent. **(07 Marks)**

c. An author has four typists typing the manuscript of his latest book. Typist A does 30% of the typing; typist B 25%; typist C 20% and typist D, 25%. Errors occur on 5% of the pages typed by A, on 4% types by B, on 3% typed by C and on 2% typed by D. If a page is chosen at random what is the probability that it contains errors? If a page chosen contains errors. what is the probability that it was typed by typist A or typist B? **(07 Marks)**

7 a. A random variable x has the density function

$$f(x) = \begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate K, and find (i) $p(1 \le x \le 2)$ ii) $p(x \le 2)$ iii) $p(2 < x \le 3)$ and iv) $p(x > 1)$. **(06 Marks**

b. Find the mean, variance and standard deviation for the binomial distribution. **(07 Marks**

c. The life of a certain type of electrical lamps is normally distributed with mean of 2040 hr and standard deviation 60 hours. In a consignment of 2000 lamps, find how many would b expected to burn for (i) more than 2150 hours (ii) less than 1950 hours, and (iii) betwee 1920 hours and 2160 hours given that A(1.5) = 0.4332, A(1.83) = 0.4664 and A(2) = 0.477 **(07 Marks**

8 a. The mean and standard deviation of marks scored by a sample of 100 students are 67.45 an 2.92. Find (i) 95% and (ii) 99% confidence intervals for estimating the mean marks of th student population. **(06 Mark**

b. Consider the sample consisting of nine numbers 45, 47, 50, 52, 48, 47, 49, 53 and 51. Th sample is drawn from a population whose mean is 47.5. Find whether the sample mea differs significantly from the population mean at 5% level of significance. **(07 Mark**

c. Fit a binomial distribution to the following data:

x_i	0	1	2	3	4	5
f_i	2	14	20	34	22	8

Test the goodness of this fit at 5% level of significance. **(07 Mark**

* * * * *